EXERCISES [MAI 5.4]

MONOTONY – MAX – MIN

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. (a) $f'(x) = 3x^2 + 1 > 0$, f increasing (b) $f'(x) = -1 - 3x^2 \leq 0$, f decreasing

(c) $f'(x) = 10x^4 + \dfrac{1}{2\sqrt{x}} > 0$, f increasing (d) $f'(x) = x - 2 > 0$, f increasing

2. (a) $f'(x) = 3x^2 + 6x - 9$

(b) $3x^2 + 6x - 9 = 0 \Leftrightarrow x = -3$ (max) $x = 1$ (min)

(c) check the graph on your GDC

3. (a) $f'(x) = 3x^2 + 6x + 3$

(b) $3x^2 + 6x + 3 = 0 \Leftrightarrow x = -1$ (neither max nor min) [called stationary point of inflexion]

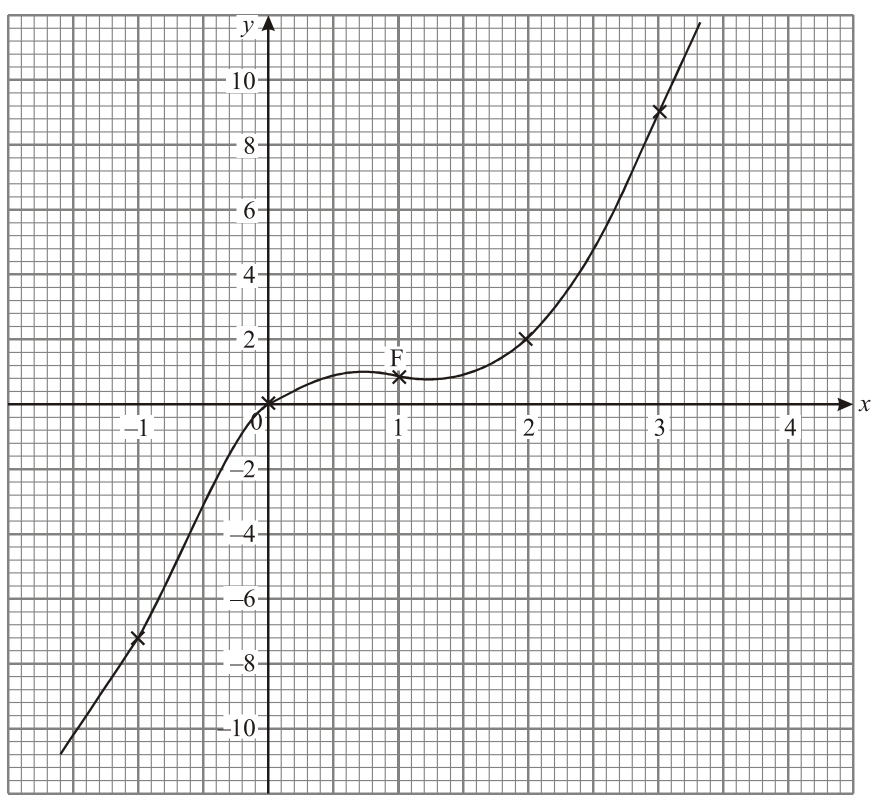
(c) check the graph on your GDC

4. (a) $f'(x) = 3x^2 - 6x + 3$

(b)

x	–1	0	1	2	3
$f(x)$	**–7**	0	1	2	9
$f'(x)$	12	**3**	0	**3**	12

(c)



(d) 12

5. (a) $g'(x) = 2px + q$

(b) $2px + q = 2x + 6$, so $p = 1$, $q = 6$

(c) (i) $g'(x) = 0 \Rightarrow 2x + 6 = 0 \Rightarrow x = -3$

(ii) $-12 = (-3)^2 + 6(-3) + c \Rightarrow -12 = 9 - 18 + c \Rightarrow c = -3$

6. (a) $g'(x) = 3x^2 + 12x + 12$

(b) $3x^2 + 12x + 12 = 0$

$x^2 + 4x + 4 = 0$

$x = -2$

(c) (i) $x = -3 \Rightarrow \dfrac{dy}{dx} = 3$ (ii) $x = 0 \Rightarrow \dfrac{dy}{dx} = 12$

(iii) (a) Increasing (b) Increasing

7. $x = 1$ (max) $x = 3$ (min)

8. $x = 1$ (max) $x = 3$ (min) $x = 5$ (neither - stationary point of inflexion)

9. (a) $g'(x) = 3x^2 - 6x - 9$

$3x^2 - 6x - 9 = 0 \Leftrightarrow 3(x - 3)(x + 1) = 0 \Leftrightarrow x = 3 \;\; x = -1$

(b) **METHOD 1**

$g'(x < -1)$ is positive, $g'(x > -1)$ is negative $\Rightarrow$ max when $x = -1$

$g'(x < 3)$ is negative, $g'(x > 3)$ is positive $\Rightarrow$ min when $x = 3$,

METHOD 2

Evidence of using second derivative

$g''(x) = 6x - 6 \Rightarrow$ max when $x = -1$

$g''(3) = 12$ (or positive),

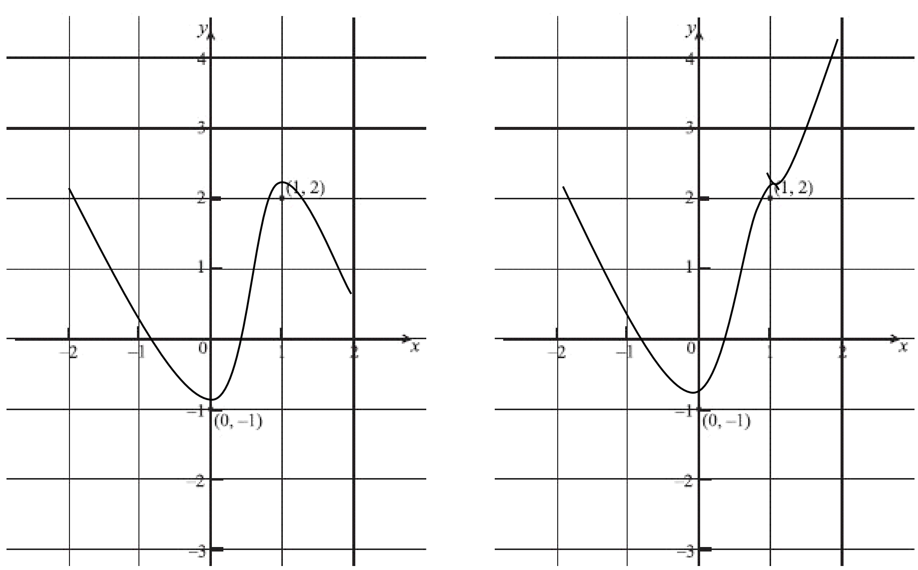
10. (a)

	A	B	C	E
$f'(x)$	negative	0	positive	negative

(b)

$f(0)$	$f'(0)$
positive	positive

11.



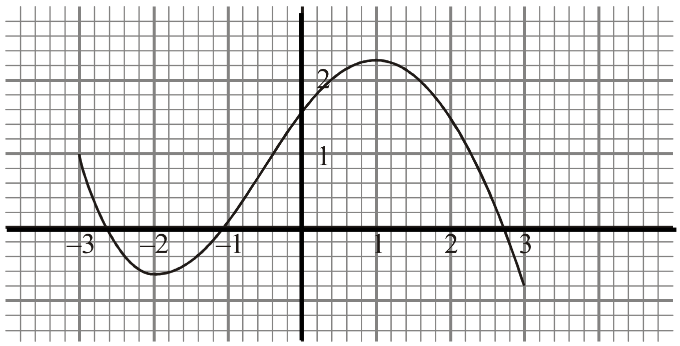
12. (a) $x = 1$

The gradient of $g(x)$ goes from positive to negative

(b) $-3 < x < -2$ **and** $1 < x < 3$

$g'(x)$ is negative

(c)



13.

	$f(x)$	$f'(x)$
$x = -3$	negative	0
$x = -2$	negative	positive
$x = -1$	0	positive
$x = 0$	positive	0
$x = 0.5$	positive	negative
$x = 2$	negative	0

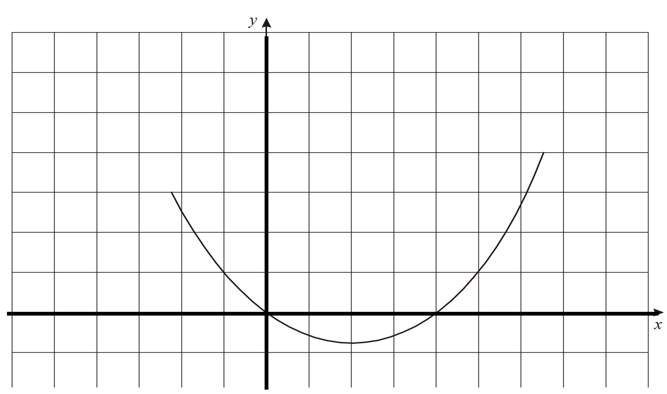
14. (a) $f'(x) = 2x - \dfrac{p}{x^2}$

(b) $f'(-2) = 0 \Leftrightarrow -4 - \dfrac{p}{4} = 0 \Leftrightarrow -\dfrac{p}{4} = 4 \Leftrightarrow p = -16$

15. (a)

x	$x < 0$	$x = 0$	$0 < x < 4$	$x = 4$	$x > 4$
$f'(x)$	positive	0	negative	0	positive

(b)



16.

Function	Derivative diagram
f_1	(d)
f_2	(e)
f_3	(b)
f_4	(a)

B. Paper 2 questions (*LONG*)

17. (a) (i) $f'(x) = 6x^2 - 6x - 12\ (+0) = 6x^2 - 6x - 12$

(ii) $f'(3) = 6(3)^2 - 6(3) - 12 = 24$

(b) $6x^2 - 6x - 12 = -12 \Rightarrow 6x^2 - 6x = 0 \Rightarrow 6x\,(x - 1) = 0 \Rightarrow x = 0$ or $x = 1$

(c) (i) $f'(x) = 0 \Rightarrow 6x^2 - 6x - 12 = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2$ or $x = -1$

(ii) $x = 2, y = -15$
Therefore, minimum is $(2, -15)$

(d) $x < -1$ and $x > 2$

18. (a) $f'(x) = 3x^2 + 12x - 15$

(b) $f'(x) = 0 \Leftrightarrow x = -5, x = 1$

(c) maximum M(-5,100), minimum M´(1,-8) $\Rightarrow$ Midpoint (-2,46)

$f(-2) = 46$

(d) $f'(-2) = -27$
$y - 46 = -27(x + 2) \Rightarrow y = -27x - 8$

19. (a) $f'(x) = 3ax^2 + 2bx + 9$

(b) $f(1) = 4 \Rightarrow a + b + 9 = 4 \Rightarrow a + b = -5$

$f'(1) = 0 \Rightarrow 3a + 2b + 9 = 0 \Rightarrow 3a + 2b = -9$

(c) $a = 1, b = -6$

(d) $f(x) = x^3 - 6x^2 + 9x$
$f'(x) = 3x^2 - 12x + 9 = 0 \Rightarrow x = 1, x = 3$

<table>
<tr><td>x</td><td colspan="2">3</td></tr>
<tr><td>$f'(x)$</td><td>–</td><td>+</td></tr>
<tr><td></td><td colspan="2">min</td></tr>
</table>

20. (a) $f'(x) = 3x^2 - 4$

(b) $f'(1) = -1$
$3x^2 - 4 = -1 \Leftrightarrow x = \pm 1$

at Q, $x = -1$, $y = 4$ (Q is $(-1, 4)$)

(c) f is decreasing when $f'(x) < 0$

$p = -1.15$, $q = 1.15$; (OR $\pm\dfrac{2}{\sqrt{3}}$)

(d) $f'(x) \geq -4$, $y \geq -4$, OR $[-4, \infty[$